RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

FIRST YEAR [2015-18]

B.A./B.Sc. FIRST SEMESTER (July – December) 2015

Mid-Semester Examination, September 2015

Date : 17/09/2015 **MATHEMATICS (General)**

Time : 12 noon – 1 pm Paper : I Full Marks : 25

Group – A

Answer **any three** : [3 × 3]

1. If $x+\frac{1}{x}=2\cos\frac{\pi}{7}$, then prove that $x^7+\frac{1}{x^7}=-2$.

2. Find the principal value of $i^{\log(1+i)}$.

3. Solve $x^4+2x^3-5x^2+6x+2=0$, given that one of the roots is $-2-\sqrt{3}$.

4. Solve $x^4-5x^3-9x^2+81x-108=0$, given that it has a multiple root.

5. If α,β,γ be the roots of $x^3-3x^2+4x-1=0$, find the value of $(\alpha+1)(\beta+1)(\gamma+1)$.

Group – B

Answer **any two** questions : [2 × 4]

6. a) For any three subsets A, B, C of the universal set U, if $A\Delta B=A\Delta C$, prove that B = C. [2]

   b) The mapping $f:\mathbb{R}\to\mathbb{R}$ is defined by $f(x)=x^2-3x+2, x\in\mathbb{R}$. Investigate that f is one-to-one or not. [2]

7. a) Find the identity element and the inverse of any element a in the group $(\mathbb{Z},*)$, where $\mathbb{Z}$ is the set of integers and '*' is defined on $\mathbb{Z}$ by $a*b=a+b+1,\ a,b\in\mathbb{Z}$. [3]

   b) Give an example to show that the union of two subgroups of a group may not be a subgroup of the group. [1]

8. a) In a ring $(\mathcal{R},\oplus,\odot)$ prove that $a\odot(-b)=-(a\odot b)\ \forall a,b\in\mathcal{R}$. [3]

   b) If $A=\{x\in\mathbb{R}:0\le x\le 3\}$ and $B=\{x\in\mathbb{R}:1<x\le 5\}$ find $(A\cap B)'$, where $U=\{x\in\mathbb{R}:0\le x\le 10\}$ is taken as the universal set. [1]

Answer **any two** questions : [2 × 4]

9. Show that the function f(x) defined as follows :

$$f(x)=\begin{cases} x\sin\frac{1}{x} & : \ x\neq 0 \\ 0 & : \ x=0 \end{cases}$$

   is continuous at x = 0. Is it derivable at x = 0? Justify your answer. [4]

10. a) Use the first principle of differentiation to find an approximate value of $\sqrt[5]{33}$. [3]

    b) Is the function f(x) = [x] monotone decreasing? [1]

11. If $y=e^{m\sin^{-1}x}$, show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+m^2)y_n=0$, where y_n denotes the n times differentiation of y. [4]

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